

Comment on the Neutrino-Mixing Interpretation of the GSI Time Anomaly

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Abstract

It is shown that neutrino mixing cannot explain the GSI time anomaly, refuting recent claims in this direction. Addendum 1: Remarks on [arXiv:0801.1465](#). Addendum 2: Quantum effects in GSI nuclear decay.

A GSI experiment observed an oscillatory time modulation of the electron-capture decay of $^{140}\text{Pr}^{58+}$ and $^{142}\text{Pm}^{60+}$ ions,

$$^{140}\text{Pr}^{58+} \rightarrow ^{140}\text{Ce}^{58+} + \nu_e, \quad ^{142}\text{Pm}^{60+} \rightarrow ^{142}\text{Nd}^{60+} + \nu_e, \quad (1)$$

with periods $T(^{140}\text{Pr}^{58+}) = 7.06(8) \text{ s}$ and $T(^{142}\text{Pm}^{60+}) = 7.10(22) \text{ s}$ [1]. The experimental collaboration wrote: “tentatively this observation is attributed to the coherent superposition of finite mass eigenstates of the electron neutrinos from the weak decay into a two-body final state” [1]. Theoretical arguments towards this interpretation have been presented in Refs. [2, 3]. Unfortunately, this interpretation is in contradiction with the well known fact that decay rates and cross sections are given by the incoherent sum over the different channels corresponding to different massive neutrinos [4, 5, 6, 7, 8, 9, 10]. In this comment I would like to explain the mistake in the calculations presented in Refs. [2, 3].

The authors of Refs. [2, 3] calculated the electron capture process using time-dependent perturbation theory with the effective time-dependent weak interaction Hamiltonian

$$H_W(t) = \frac{G_F}{\sqrt{2}} V_{ud} \int d^3x \bar{\psi}_n(x) \gamma^\mu (1 - g_A \gamma^5) \psi_p(x) \sum_{j=1}^3 U_{ej}^* \bar{\psi}_{\nu_j}(x) \gamma_\mu (1 - \gamma^5) \psi_e(x), \quad (2)$$

using standard notations. They interpreted (see Eq. (3) in Ref. [2] and Eq. (23) in Ref. [3])

$$A(t) = \sum_k A_k(t) \quad (3)$$

as the time-dependent amplitude of the decay

$$I_i \rightarrow I_f + \nu_e, \quad (4)$$

where

$$A_k(t) = \int_0^t d\tau \langle I_f, \nu_k | H_W(\tau) | I_i \rangle \quad (5)$$

is the time-dependent amplitude of

$$I_i \rightarrow I_f + \nu_k \quad (6)$$

transitions. Here I_i is the initial ion ($^{140}\text{Pr}^{58+}$ or $^{142}\text{Pm}^{60+}$), I_f is the final ion ($^{140}\text{Ce}^{58+}$ or $^{142}\text{Nd}^{60+}$, respectively), and ν_k are the massive neutrinos ($k = 1, 2, 3$).

Regrettably, the amplitude in Eq.(3) does not describe the decay (4), but a decay in which the final neutrino state is

$$|\nu\rangle = \sum_k |\nu_k\rangle, \quad (7)$$

which is clearly different from an electron neutrino state. Indeed, in the standard theory of neutrino oscillations (see Refs. [11, 12, 13, 14, 15, 9, 10]) electron neutrinos are described by the state

$$|\nu_e\rangle = \sum_k U_{ek}^* |\nu_k\rangle, \quad (8)$$

where U is the unitary mixing matrix of the neutrino fields in Eq. (2). More accurately, if the neutrino mass effects in the interaction processes are taken into account [16, 17, 15, 18, 9, 10], in the time-dependent perturbation theory used in Refs. [2, 3] the final electron neutrino in the process (4) is described by the normalized state

$$|\nu_e(t)\rangle = \left(\sum_j |A_j(t)|^2 \right)^{-1/2} \sum_k A_k(t) |\nu_k\rangle. \quad (9)$$

The time dependence of this electron neutrino state takes into account the fact that in time-dependent perturbation theory the final state of a process is studied during formation.

Using the correct electron neutrino state in Eq. (9), the decay amplitude is not given by Eq.(3), but by

$$A(t) = \left(\sum_j |A_j(t)|^2 \right)^{-1/2} \int_0^t d\tau \sum_k A_k^*(t) \langle I_f, \nu_k | H_W(\tau) | I_i \rangle = \left(\sum_k |A_k(t)|^2 \right)^{1/2}. \quad (10)$$

Then, it is clear that the electron capture probability is given by the incoherent sum over the different channels of massive neutrino emission. In other words, there is no interference term between different massive neutrinos contributing to the rates of the electron capture processes in Eq. (1), as well as all decay rates and cross sections.

In conclusion, I have shown that neutrino mixing cannot explain the GSI time anomaly [1], refuting the claims presented in Refs. [2, 3].

Addendum 1: Remarks on arXiv:0801.1465

This addendum is motivated by associations of Ref. [19] with the GSI anomaly and the first version of this comment (see for example the January 2008 issue of *Long-Baseline Neutrino Oscillation Newsletters* at www.hep.anl.gov/ndk/longbnews/0801.html).

The author of Ref. [19] predicted¹ the possibility to observe oscillating decay probabilities due to neutrino mixing.

Frankly speaking, the arguments presented in Ref. [19] seem to me rather obscure. Furthermore, no real calculation of the decay probability is presented. Therefore, I wish only to comment on the points in Ref. [19] which are relevant for the problem under discussion:

1. At the beginning of Section II.B of Ref. [19]: “Both energy and momenta are conserved for each component of the wave packet which has a momentum \vec{P} and energy E in the initial state.”

There is no such conservation law.

In a scattering or decay process with particles described by plane waves, the energies and momenta of all the particles have definite values. The total energy and momentum are exactly conserved. Mathematically, the conservation follows from the integration over space-time of a product of plane waves which generates an energy-momentum delta function. Obviously this is an approximation (very often very good).

In reality, since all processes are localized in space-time, there is always an energy-momentum uncertainty. In this case, the interacting particles are described by wave packets. Talking of exact energy-momentum conservation makes no sense, since energy and momentum do not have definite values. They are conserved only within their uncertainty. Mathematically, the energy-momentum delta function is replaced by a factor which suppresses energy-momentum violations larger than the energy-momentum uncertainty (see Refs.[20, 21]).

Notice that in this case nothing can be said about the behavior of each component of a wave packet. A wave packet interacts as a whole.

2. Before Eq. (2.4) of Ref. [19]: “The difference in momentum δp_ν between the two neutrino eigenstates with the same energy produces a small initial momentum change $\delta P \dots$ ”.

This would be a violation of causality: final states are determined by initial states, not vice versa.

In any case, what is the meaning of δP if the initial state is described by a wave packet which depends on the way in which the initial state has been prepared?

3. The relevant discussion in Section II.C of Ref. [19] is based on δP , which is physically meaningless.

Addendum 2: Quantum effects in GSI nuclear decay

In the first version of this addendum I incorrectly claimed that the GSI anomaly cannot be due to a quantum effect in nuclear decay. I would like to thank Yu.A. Litvinov for

¹ Ref. [19] is dated 9 January 2008, but the work started in 2006 (see the Acknowledgments).

an enlightening discussion on this point at the IV International Workshop on: “Neutrino Oscillations in Venice” (15-18 April 2008, Venice, Italy).

Although the ions are monitored with a frequency of the order of the revolution frequency in the ESR storage ring, about 2 MHz, the GSI anomaly could be due to the quantum interference between two coherent states of the decaying ion if the interaction with the measuring apparatus does not distinguish between the two states. In order to produce quantum beats with the observed period of about 7 s, the energy splitting between the two states must be of the order of 10^{-16} eV. It is very likely that the measuring apparatus which monitors the ions in the ESR storage ring does not distinguish between these two states and their coherence is preserved for a long time.

The problem is to find the origin of such a small energy splitting. The authors of Ref. [1] noted that the splitting of the two hyperfine $1s$ energy levels of the electron is many order of magnitude larger (and the contribution to the decay of one of the two states is suppressed by angular momentum conservation). It is difficult to find a mechanism which produces a smaller energy splitting.

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